Math 10 (200 Points) -- ANSWERS

Final Exam

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[1] Completely factor in the real numbers $(p-q)^2 - 4r^2$.
ANS: $(p-q-2r)(p-q+2r)$

[2] Completely factor in the real numbers $x^4 - 16$.
ANS: $(x-2)(x+2)(x^2+4)$

[3] Completely factor in the real numbers $x^4 - 6x^2 + 9$.
ANS: $(x^2-3)^2$

[4] Completely factor in the real numbers $x^2 + x - y^2 - y$.
ANS: $(x-y)(x+y+1)$

[5] Find all real and complex roots. $(x^2-x)^2 - 4(x^2-x) - 12 = 0$
ANS: $\left\{\{x \to -2\}, \{x \to 3\}, \left\{x \to \frac{1}{2} - \frac{i\sqrt{7}}{2}\right\}, \left\{x \to \frac{1}{2} + \frac{i\sqrt{7}}{2}\right\}\right\}$

[6] Find all real and complex roots. $x^2 - 4x + 1 = 0$.
ANS: $\left\{\left\{x \to 2 - \sqrt{3}\right\}, \left\{x \to 2 + \sqrt{3}\right\}\right\}$

[7] Find all real and complex roots. $2x^2 - 11x + 15 = 0$.
ANS:$\{\{x \to \frac{5}{2}\}, \{x \to 3\}\}$

[8] Find the value of p such that $x^3 - 3x^2 - x + p$ is divisible by 3.
ANS: $x = 3$

[9] Find all cube roots of unity; that is, solve in the complex numbers the equation $x^3 = -1$.
ANS: $\left\{\{x \to -1\}, \left\{x \to \frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}, \left\{x \to \frac{1}{2} - \frac{i\sqrt{3}}{2}\right\}\right\}$

[10] Simplify. $\sqrt{-2}\,\sqrt{-2}$.
ANS: -2

[11] State the broadest conditions on a and b under which it must be true that $a > b \Longrightarrow a^2 > b^2$.
ANS: $a > 0, b > 0$

[12] Find all real and complex solutions of $1 + 2x - x^2 \geq 0$.
ANS: $1 - \sqrt{2} \leq x \leq 1 + \sqrt{2}$

[13] Find all real and complex solutions of $x^2 + 2x + 2 = 0$.
ANS: $\{\{x \to -1 - i\}, \{x \to -1 + i\}\}$

[14] Find all real and complex solutions of $x^3 - 2x^2 - x + 2 = 0$.
ANS: $\{\{x \to -1\}, \{x \to 1\}, \{x \to 2\}\}$

[15] Find the equation of the line through $P(-2, 3)$, $Q(5, 9)$. Answer in standard form.
ANS: $6x - 7y = -33$

[16] Find the equation of the line through $P(-1, -5)$ and perpendicular to $y = -\frac{1}{3}x + 100$. Answer in standard form $a\,x + b\,y + c = 0$.
ANS: $3\,x - y = 2$

[17] Find the equation of the line(s) through $P(1, 5)$ and tangent to the circle $x^2 + y^2 = 1$.
ANS: $\ell_1 : -12\,x + 5\,y = 13,$ and $\ell_2 : x = 1$.

[18] Find the center and radius of the circle $x^2 + 4\,x + y^2 - 6\,y + 9 = 0$.
ANS: $C(-2, 3)$, $r = 2$

[19] Find the maximum value (if it exists) and the minimum value (if it exists) of each of the function $y = x^2 - 5\,x + 6,\ x \le 4$.
ANS: The minimum value of y is $\frac{-49}{4}$. The maximum value of y is 2.

[20] Find the quadratic function that passes through $P(3, 7)$ and whose vertex is $V(2, 5)$.
ANS: $y = 2\,(x - 2)^2 + 5$